arXiv:cond-mat/0410193 v1 8 Oct 2004

Instability toward biexciton crystallization in one-dimensional electron-hole systems

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Abstract

One-dimensional (1D) electron-hole (e-h) systems in a high-density regime is investigated by means of bozonization techniques. It turned out that the systems are insulating even at the high density limit and that the exciton Mott transition (insulator-to-metal transition) never occurs at absolute zero temperature. The insulating ground state exhibits a strong instability towards the crystallization of biexcitons.

Key words: electron-hole system, exciton Mott transition, one-dimension, Tomonaga-Luttinger liquid, bosonization
PACS: 71.35.-y, 71.35.Ee, 71.10.Pm

1. Introduction

For more than three decades, electron-hole (e-h) systems realized in strongly photo-excited semiconductors have been intensively studied both theoretically and experimentally. One of keywords to understand these systems is the exciton Mott transition (insulator-to-metal transition), which has been believed to take place with increasing of photo-excitation (e-h density). The main origin of this transition is enhancement of the screening effects which weaken the binding energy of the e-h bound states [1]. Such exciton Mott transition is confirmed both theoretically and experimentally only in three-dimensional (3D) systems.

Quite recently, extremely clean one-dimensional (1D) e-h systems are realized experimentally in quantum wires fabricated in the T-shaped quantum wells [2]. From the theoretical point of view, such 1D systems are highly special since at 1D an electron and a hole form an exciton bound state even when the attractive e-h interaction is infinitesimally weak. Therefore, we naively expect that the exciton Mott transition is absent in 1D. This argument above, however, is insufficient since the dynamical screening effects are not considered.

Some theoretical proposals on the topic of the exciton Mott transition in 1D has been reported in Ref.[3], where a Bethe-Salpeter equation is solved approximately. They found a critical e-h density, n_c, where the exciton bound state is dissociated, at absolute zero temperature even in 1D. Nevertheless, the calculated optical absorption spectra show no evidence of the exciton Mott transition,

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i.e., the exciton peak survives even for densities higher than n_c. They claimed that this nonvanishing exciton peak explains some experiments [4]. Their results are, however, unreliable because their perturbation theory based on the Fermi liquid picture, which is not applicable to the 1D systems.

With increasing the e-h density, the system goes to the weak-coupling regime, where the characteristic interaction energy is much smaller than the Fermi energies of electrons and holes. In this regime, the bosonization method has advantages in that it can take full account of the interaction processes. The first application of this method to the 1D e-h systems is given in Ref.[5], in which electrons and holes are assumed to interact with each other through short-range force. In the following, we apply this technique to the 1D e-h system with long-range Coulomb interaction.

Now, we turn to the properties of the insulating phase. In 1D systems, the possibility of exciton crystallization is theoretically pointed out [6] in the spinless e-h systems at intermediate e-h densities, which is not expected to take place in 3D systems. Here, "crystallization" means that the excitons are located at regular intervals. In the following, we will discuss the possibility of the biexciton crystallization in 1D systems consisting of electrons and holes with spin at absolute zero temperature.

2. Formulation

In the following, we consider properties of the "ground state" of a one-dimensional e-h system. Such consideration makes sense in comparison with the actual experimental situation because the interband relaxation time is usually long enough to realize intraband thermal quasiequilibrium.

In the weak-coupling regime, the low-energy physics is unaffected by the linearization of band dispersion, $\epsilon^{(\mu)}(k) = r v_{\rm F}^{(\mu)}(k - r k_{\rm F})$, near the right $(r = +)$ and the left $(-)$ Fermi points $k = r k_{\rm F}$, respectively, where $v_{\rm F}^{(\mu)}$ denotes the Fermi velocity and $\mu = {\rm e}, {\rm h}$ is the index for the electron and the hole, respectively. When both electron and hole bands are far off the half filling, the umklapp processes are negligible. Then, the Hamiltonian can be written in the g-ology form, which is characterized by the momentum-dependent coupling parameters $g_1^{(\mu\mu')}(q)$, $g_2^{(\mu\mu')}(q)$ and $g_4^{(\mu\mu')}(q)$. Here, $g_1^{(\mu\mu')}$ specifies the backward scattering, while $g_2^{(\mu\mu')}$ and $g_4^{(\mu\mu')}$ do the forward ones. Note that there is interband (e-h) scattering process $(\mu \neq \mu')$ as well as the intraband (e-e and h-h) scattering process $(\mu = \mu')$. For simplicity, we restrict ourselves to the e-h symmetric case, $v_{\rm F} = v_{\rm F}^{({\rm e})} = v_{\rm F}^{({\rm h})}$ and $g_n^{({\rm ee})} = g_n^{({\rm hh})}$. The conclusions obtained below are quite general and qualitatively unchanged for the asymmetric cases. More detailed and general calculations will be presented elsewhere [7].

In the quantum wire formed in T-shaped quantum wells, the interaction potential behaves as unscreened Coulomb potential $(\propto 1/r)$ at the long interparticle distance r. Thus, the coupling parameters of the forward scattering show a logarithmic divergence at $q \to 0$ [8] In fact, they can be approximated in the form of $g_{2,4}^{({\rm ee})}(q) = g_{2,4}^{({\rm hh})}(q) \sim g_{2,4} + 2g_0 K_0(|q|d)$ and $g_{2,4}^{({\rm eh})}(q) = g_{2,4}^{({\rm he})}(q) \sim -g'_{2,4} - 2g_0 K_0(|q|d)$, using the modified Bessel function $K_0(x)$, and the effective diameter of the quantum wire d. The coupling constants $g_{2,4}$ and $g'_{2,4}$ specify the short-range part of forward scattering, and $g_0 = e^2/\epsilon\pi v_{\rm F}$ does its long-range part, where e and ϵ denote the electron charge and the dielectric constant, respectively. On the other hand, the coupling parameters of the backward scattering can be substituted to constants as $g_1^{({\rm ee})}(q) = g_1^{({\rm hh})}(q) = g_1$ and $g_1^{({\rm eh})}(q) = g_1^{({\rm he})}(q) = -g'_1$, because they show no singularity at $q \sim \pm 2k_{\rm F}$.

Now, we introduce the phase operators $\Phi_\nu^{(\xi)}(x)$ and $\Theta_\nu^{(\xi)}(x)$ $(\xi = \pm,\ \nu = \rho, \sigma)$ through

$$\partial_x \Phi_\rho^{(\pm)} = -\pi \sum_{r\sigma} \rho_{r\sigma}^{(\pm)},\ \partial_x \Phi_\sigma^{(\pm)} = -\pi \sum_{r\sigma} (-1)^{\delta_{\sigma\downarrow}} \rho_{r\sigma}^{(\pm)},$$

$$\partial_x \Theta_\rho^{(\pm)} = \pi \sum_{r\sigma} r \rho_{r\sigma}^{(\pm)},\ \partial_x \Theta_\sigma^{(\pm)} = \pi \sum_{r\sigma} r (-1)^{\delta_{\sigma\downarrow}} \rho_{r\sigma}^{(\pm)},$$

where $\sigma = \uparrow, \downarrow$ denotes the spin, $\delta_{\sigma\sigma'}$ is the Kronecker's delta, $\rho_{r\sigma}^{(\pm)} = (\rho_{r\sigma}^{({\rm e})} \pm \rho_{r\sigma}^{({\rm h})})/2$, and $\rho_{r\sigma}^{(\mu)}(x)$ denotes the density at position x for the particle with indices r and σ. It is noteworthy that $\Phi_\rho^{(+)}$ and $\Phi_\rho^{(-)}$ are associated with the mass and charge densities (the sum and difference of electron and

hole densities), respectively.

Then, the g-ology Hamiltonian is written as

$$\mathcal{H} = \sum_{\nu=\rho,\sigma,\xi=\pm} \mathcal{H}_\nu^{(\xi)} + \mathcal{H}_{\rm bs}^{(\rm intra)} + \mathcal{H}_{\rm bs}^{(\rm inter)},$$

$$\mathcal{H}_\nu^{(\xi)} = \frac{v_\nu^{(\xi)}}{2\pi}\int dx \left\{ K_\nu^{(\xi)}\left[\partial_x \Theta_\nu^{(\xi)}\right]^2 + \frac{1}{K_\nu^{(\xi)}}\left[\partial_x \Phi_\nu^{(\xi)}\right]^2 \right\}$$
$$+ \frac{v_{\rm F}}{\pi}\int dx dx' \frac{2g_0 \delta_{\xi-}\delta_{\nu\rho}}{\sqrt{(x-x')^2+d^2}} \partial_x \Phi_\rho^{(-)} \partial_{x'} \Phi_\rho^{(-)},$$

$$\mathcal{H}_{\rm bs}^{(\rm intra)} = \frac{v_{\rm F} g_1}{\pi\alpha^2}\int dx \cos 2\Phi_\sigma^{(+)} \cos 2\Phi_\sigma^{(-)},$$

$$\mathcal{H}_{\rm bs}^{(\rm inter)} = -\frac{v_{\rm F} g_1'}{\pi\alpha^2}\int dx \sum_\xi \cos 2\Phi_\rho^{(-)} \cos 2\Phi_\sigma^{(\xi)} \qquad (1)$$

Here, we used the cut-off constant α, the renormalized velocity $v_\nu^{(\xi)} = v_{\rm F}(a_\nu^{(\xi)} b_\nu^{(\xi)})^{1/2}$ and coupling constant $K_\nu^{(\xi)} = (a_\nu^{(\xi)}/b_\nu^{(\xi)})^{1/2}$, where $a_\nu^{(\xi)}$ and $b_\nu^{(\xi)}$ are given as $a_\rho^{(\pm)} = 1 + g_4 - g_2 + g_1/2 \mp g_4' \pm g_2'$, $b_\rho^{(\pm)} = 1 + g_4 + g_2 + g_1/2 \mp g_4' \mp g_2'$, $a_\sigma^{(\pm)} = 1 + g_1/2$, and $b_\sigma^{(\pm)} = 1 - g_1/2$.

The field operators for the electron and the hole can also be expressed as $\psi_{r\sigma}^{(\mu)}(x) = (2\pi\alpha)^{-1/2}\eta_{r\sigma}^{(\mu)} \exp[ir(k_{\rm F}x + \phi_{r\sigma}^{(\mu)})]$ in terms of the Klein factor $\eta_{r\sigma}^{(\mu)}$ and the phase operators, $\phi_{r\sigma}^{(\rm e)} = \sum_{\xi=\pm}(-\Phi_\rho^{(\xi)} - \sigma\Phi_\sigma^{(\xi)} + r\Theta_\rho^{(\xi)} + r\sigma\Theta_\sigma^{(\xi)})/2$ and $\phi_{r\sigma}^{(\rm h)} = \sum_{\xi=\pm}\xi(-\Phi_\rho^{(\xi)} - \sigma\Phi_\sigma^{(\xi)} + r\Theta_\rho^{(\xi)} + r\sigma\Theta_\sigma^{(\xi)})/2$.

3. Results

If we neglect the intra- and interband backward scattering terms, $\mathcal{H}_{\rm bs}^{(\rm intra)}$ and $\mathcal{H}_{\rm bs}^{(\rm inter)}$, the Hamiltonian is decoupled into four parts, each reduced to the Tomonaga-Luttinger (TL) form. In this case, they can be exactly diagonalized and the system is shown to have four gapless excitation modes corresponding to four different degrees of freedom: $(\nu,\xi) = (\rho,+)$, $(\rho,-)$, $(\sigma,+)$, and $(\sigma,-)$.

We can also evaluate the asymptotic behavior of the various correlation functions with the form, $C(x) = \langle \mathcal{O}^\dagger(x)\mathcal{O}(0)\rangle$. After some calculations similar to those of Ref.[8], we find that the correlation function of $\mathcal{O}_{\rm BED} = \psi_{+\uparrow}^{(\rm e)\dagger}\psi_{+\downarrow}^{(\rm e)\dagger}\psi_{-\downarrow}^{(\rm h)\dagger}\psi_{-\uparrow}^{(\rm h)\dagger}\psi_{+\uparrow}^{(\rm h)}\psi_{+\downarrow}^{(\rm h)}\psi_{-\downarrow}^{(\rm e)}\psi_{-\uparrow}^{(\rm e)} + {\rm h.c.} \sim \cos 4\Phi_\rho^{(-)}$ show the slowest decay. It is interesting to note that $\mathcal{O}_{\rm BED}$ appears in the slowly varying ($0k_{\rm F}$) component of the biexciton density (BED) operator. The biexciton creation operator here is given as $\psi_\uparrow^{(\rm e)\dagger}\psi_\downarrow^{(\rm e)\dagger}\psi_\downarrow^{(\rm h)\dagger}\psi_\uparrow^{(\rm h)\dagger}$ with $\psi_\sigma^{(\mu)\dagger} = \sum_r \psi_{r\sigma}^{(\mu)\dagger}$. This fact shows the strong tendency toward the biexciton formation. In this sense, the ground state has the character of "biexciton liquid."

Now, we turn to effects of the backward scattering terms. It is noteworthy that intraband backward scattering is (marginally) irrelevant if the interband one is absent. Thus, we can neglect it to investigate the relevancy of the interband backward scattering. We treat the interband backward scattering term in the self-consistent harmonic approximation (SCHA) and the renormalization group (RG) method. The both results show that the interband backward scattering is always relevant independent of $g_0 > 0$. Because the coupling constant g_1' is renormalized to a large value, $\Phi_\rho^{(-)}$ and $\Phi_\sigma^{(\pm)}$ are fixed by the condition: $\cos 2\Phi_\rho^{(-)} = \cos 2\Phi_\sigma^{(+)} = \cos 2\Phi_\sigma^{(-)} = \pm 1$, which minimizes the interband backward scattering term.

This localization of the phases leads to the energy gaps in the excitation modes corresponding to $(\nu,\xi) = (\rho,-)$, $(\sigma,+)$, and $(\sigma,-)$. The point is that the charge excitation, $(\nu,\xi) = (\rho,-)$, is gapful. This results in that the system is an insulator even in the high e-h density limit, and suggests that the exciton Mott transition is absent at absolute zero temperature.

There remains a single gapless excitation corresponding to $(\nu,\xi) = (\rho,+)$, since $\mathcal{H}_\rho^{(+)}$ is decoupled from the other part of the Hamiltonian. As will be mentioned below, its coupling constant $K_\rho^{(+)}$ specify the power of the algeblaic decay of some correlation functions.

Now, let us discuss the character of the insulating ground state obtained above. To this end, we investigate the asymptotic behavior of the correlation functions $C(x) = \langle \mathcal{O}^\dagger(x)\mathcal{O}(0)\rangle$. There are two important points.

1. Because the phases $\Phi_\rho^{(-)}$ and $\Phi_\sigma^{(\pm)}$ are localized, the fluctuations of their conjugate operators $\Theta_\rho^{(-)}$ and $\Theta_\sigma^{(\pm)}$ diverge. Thus, $C(x)$ decays

exponentially if $\mathcal{O}(x)$ contains $\Theta_\rho^{(-)}$ or $\Theta_\sigma^{(\pm)}$.

2. If $\mathcal{O}(x)$ vanishes under the phase-fixing condition, $C(x)$ shows an exponential decay.

Owing to these criteria, it is sufficient to consider the following two operators, $\mathcal{O}_{\rm MDW} = \sum_{\mu r\sigma} \psi_{r\sigma}^{(\mu)\dagger}\psi_{-r\sigma}^{(\mu)} \sim \cos(2k_{\rm F}x - \Phi_\rho^{(+)})$ and $\mathcal{O}_{\rm BEI} = \psi_{r\uparrow}^{(\rm e)}\psi_{r\downarrow}^{(\rm e)}\psi_{-r\downarrow}^{(\rm h)}\psi_{-r\uparrow}^{(\rm h)} \sim \exp(2i\Theta_\rho^{(+)})$, where these expressions are derived using the condition $\cos 2\Phi_\rho^{(-)} = \cos 2\Phi_\sigma^{(+)} = \cos 2\Phi_\sigma^{(-)} = 1$.

The operator $\mathcal{O}_{\rm MDW}(x)$ denotes the $2k_{\rm F}$ oscillatory component of mass density wave (MDW), while $\mathcal{O}_{\rm BEI}(x)$ annihilates a "biexciton" located at x. The asymptotic forms of their correlation functions can be explicitly evaluated as $C_{\rm MDW}(x) \sim x^{-K_\rho^{(+)}/2}$ and $C_{\rm BEI}(x) \sim x^{-2/K_\rho^{(+)}}$. As a result, we can see that the formation of the $2k_{\rm F}$-MDW and the Bose-Einstein condensation of biexcitons (biexcitonic insulator, BEI), shows the strongest instability at $K_\rho^{(+)} < 2$ and $K_\rho^{(+)} > 2$, respectively.

In the weak coupling regime ($g_n \ll 1$ and $g_n' \ll 1$), we obtain $K_\rho^{(+)} \sim 1$, which leads to the strong instability toward the formation of the $2k_{\rm F}$-MDW. The formation of $2k_{\rm F}$-MDW can be interpreted as "biexciton crystallization." In fact, the $2k_{\rm F}$ charge density waves (CDWs) of the electron and the hole are simultaneously formed in an in-phase way in the $2k_{\rm F}$-MDW state since the fluctuation of the total charge density is strongly suppressed. As a result, two electrons and two holes are effectively accumulated to form "biexcitons" arranged regularly with the periodicity $\pi/k_{\rm F}$.

4. Summary and Discussion

By means of the SCHA and RG techniques, we examined the relevancy of the interband backward scattering, and showed that it is always relevant and the charge gap Δ opens. This means that the system is an insulator even in the high e-h density regime in 1D, and indicates the absence of the exciton Mott transition. We also investigated the character of this insulator and found the strong instability toward the biexciton crystallization.

Our results clearly show the importance of biexciton correlation, whereas the previous theories take into account only the exciton correlations. It was also shown that only the collective modes are significant, that is the special feature of the 1D system. This fact also indicates that the traditional criterion for the exciton Mott transition is not applicable to the 1D e-h system, in which the formation of the bound state between a single electron and a single hole is considered using screened interaction [3].

In experiments, we can expect that the temperature $k_{\rm B}T$ is larger than the energy gap Δ. In this case, the effects of the backward scattering terms are negligible. Then, the system shows the character of "biexciton liquid," within the length scale $\sim v_{\rm F}/k_{\rm B}T$. This result is quite contrast to that for the e-h TL model with short-range interaction [5], in which the $2k_{\rm F}$-CDW of the hole is the most dominant order. The "biexciton-liquid" character is consistent with the recent observation of the photoluminescence spectra [2]; they show a broad peak at high e-h density, which continuously connects with the biexciton peak at low e-h density.

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